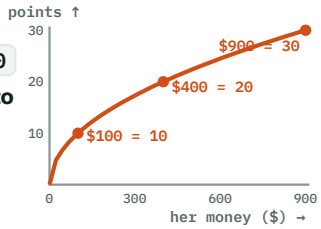
Would you take the coin flip?

Nadia has $400. A coin flip gives $900 or $100: $500 on average. Many people still keep the $400, and they are not wrong: dollars feel like points that grow more and more slowly, so a risk has a price. Insurance is the same flip read backwards.

1 Dollars → points

Bernoulli, 1738

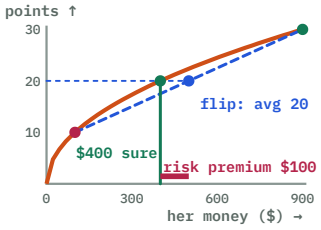
- **Expected value** = average result if the bet were repeated many times · flip `½ × 900 + ½ × 100 = $500` > $400 sure
- **Points** (made up, like Micro 11's lunch points) · `$0 = 0 · $100 = 10 · $400 = 20 · $900 = 30 · $1,600 = 40 · $2,500 = 50`
- Each 10 more points cost **more dollars**: $100 → $300 → $500 ⇒ **diminishing returns to money** = Micro 11's falling marginal utility (extra points from one more unit), for money
- Daniel **Bernoulli**, Swiss mathematician, **1738** ⇒ an extra dollar counts less the more you have



2 Average the points

expected utility

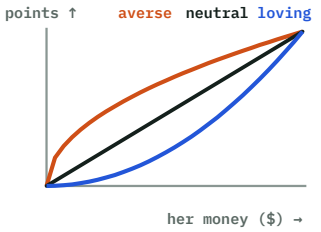
- **Expected utility** = average of the **points**, not of the dollars ⇒ `½ × 30 + ½ × 10 = 20` · von **Neumann** (Hungarian-American mathematician) and **Morgenstern** (Austrian economist), **1944**
- **Certainty equivalent** = sure amount with the same points ⇒ 20 points = **$400**
- **Risk premium** = expected value – certainty equivalent ⇒ `$500 − $400 = $100` ⇒ keeping $400 is **not a mistake**: it comes from the bend
- Practice ⇒ $1,600 or $400: avg **$1,000**, 30 pts, sure **$900**, premium **$100** · wider, $2,500 or $100: avg **$1,300**, 30 pts, sure $900, premium **$400** ⇒ results further apart ⇒ dashed line lower ⇒ **bigger premium**



3 Three attitudes

the shape of the curve

- **Risk-averse** = curve bends **down** ⇒ values a bet **below** its average ⇒ pays to avoid risk (takes $450 sure for a $500 flip · insures above the expected loss)
- **Risk-neutral** = **straight** line ⇒ only the average counts ⇒ premium $0 (a huge insurer)
- **Risk-loving** = bends **up** ⇒ values a bet **above** its average ⇒ pays to take risk ($2 ticket worth $1 on average)
- **Friedman** and **Savage**, American economists, **1948** ⇒ many people insure **and** buy lottery tickets ⇒ a curve that bends both ways



4 Insurance

the flip, read backwards

- Scooter $800 + $100 cash = $900 · made-up risk: 1 in 2 stolen ⇒ $900 or $100 = the same flip
- **Premium** = price of insurance · **fair premium** = expected loss ⇒ `½ × $800 = $400`
- **Full insurance** = same money either way ⇒ pay $500 ⇒ $400 sure = 20 pts ⇒ most she would pay **$500** ⇒ $500 – $400 = **$100** = risk premium = room for the insurer's costs ⇒ real premiums a little above fair
- Practice · 1 in 4 of losing $2,400 from $2,500 ⇒ fair `¼ × 2,400 = $600` · points `(50+50+50+10) ÷ 4 = 40` = $1,600 sure ⇒ most **$900** · risk premium **$300**
- **Law of large numbers** ⇒ many separate scooters average out ⇒ 100 scooters: ~25 stolen ⇒ $60,000 ⇒ **$600** each ⇒ a gamble for one owner, an almost sure cost for the insurer

5 Limits and next

idea 4 of 11

- People weigh a **loss** more than an equal gain ⇒ **loss aversion** (Behavioral 3) ⇒ left out by this rule
- John **Pratt** (statistician) and Kenneth **Arrow** (economist), American, **1964-65** ⇒ measured how fast the curve bends ⇒ more bend = bigger premium ⇒ university: the **Arrow–Pratt coefficient**
- **Eleven ideas behind the curves** · `1 · A demand curve is a chain of best bundles` `2 · A price change is two effects: substitution and income` `3 · Selling what you own is the same choice` `4 · Risk has a price: the risk premium`

Watch out: average the **points**, not the dollars · a risk-averse choice is **not irrational** ($100 is the price of being sure) · fair premium = expected **loss**, not the whole loss ($400, not $800)

This is a lesson, not financial advice. It shows what the evidence says about choices like these; what is right for you depends on your life.