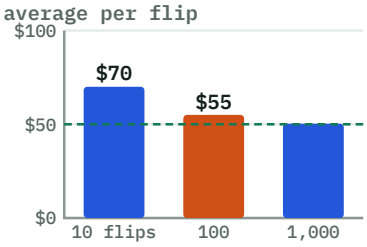
How likely is it?

A **probability** = share of the equally likely ways · **expected value** = each outcome × its chance, added up · = the **average over many tries**, not a prize you win once · many tries land near it, a few can land far. Tool 10 of the eleven.

1 Many flips

Lia draws

- **Lia** (draws) + **Sam** (formulas) · **Coralia** (toy island, dollars) · coins, dice and toy prices only: no real bet, lottery or insurer, nothing is advice
- Coin flip: heads **$100**, tails **$0** · Lia flips again and again → average prize per flip
- 10 flips: 7 heads → **$70** · 100: 55 heads → $55 · 1,000: 503 heads → **$50.30** → settles on the dashed line, **$50**
- **Probability** = equally likely ways that give it ÷ all the ways · coin heads `1 in 2 = ½` · die six `1 in 6 = ⅙` · die 5 or 6 `2 in 6 = ⅓` · 0 (never) to 1 (always) · **all** the chances add up to **1**



2 The formula

Sam writes · E[X]

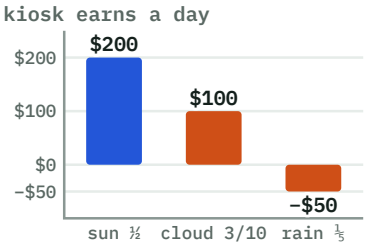
- **X** = the prize, unknown until the coin lands · **E[X]** = its **expected value** (or mean) · **P** = a probability · `E[X] = ½ × $100 + ½ × $0 = $50` · Lia: "worth $100" = the prize · Sam: "$50" = the value · both right
- Toy ticket: $2, `1 in 1,000 × $1,000 = $1` → the seller keeps $1 of every $2 on average
- Die, $1 a dot: `⅙ × (1 + 2 + … + 6) = 21 ÷ 6 = $3.50` · no face pays $3.50: it is the average · six rolls ≈ one of each face = $21
- Raffle, 100 tickets at $3: one $100 → `1/100 × 100 = $1` · four $25 → `4/100 × 25 = $1` → **E = $2** · club keeps $1 a ticket



3 Losses and spread

same tool, more outcomes

- **Insurance** (Personal Finance 11): 1 in 100 to lose $10,000 → expected loss **$100** = **fair premium** (premium = the price of insurance) · a real premium a little more (insurer's costs)
- A **loss = negative prize** · Amara's fertilizer (Development 6): `¾ × 20 + ¼ × (−20) = 15 − 5 = 10` pesos (Palmera's money) · new seed: `⅘ × 30 + ⅕ × (−50) = 24 − 10 = 14` (not 34)
- **Three outcomes**, same rule · kiosk: sun ½ × $200 + clouds 3/10 × $100 + rain ⅕ × (−$50) → **$120** a day · rain = 1 − ½ − 3/10 = ⅕
- **Spread** = how far results land from the average · sure 10 pesos vs flip 0 or 20: same average, different spread · **variance** (university name) · why Amara hesitates



4 Fair, loses, wins

price vs E

For the one who pays	Rule	Examples (price → E)
Fair	price = E	$50 for the $100 flip → $50 · $100 insurance, 1 in 100 of $10,000 → $100 · $3 die, $18 on a six → $3 · $40 insurance, 1 in 50 of $2,000 → $40
Loses on average	price > E	$2 ticket → $1 · $1 for 1 in 10 of $5 → $0.50 · $1 for 1 in 4 of $2 → $0.50 · $10 for 1 in 5 of $40 → $8
Wins on average	price < E	$20 for the $100 flip → $50 · $10 for 1 in 2 of $30 → $15 · $5 flip of $20 → $10 · $2 for 1 in 3 of $9 → $3

5 So what?

tool 10 of 11

- **Law of large numbers**: many tries → the average lands **near** E (not exactly) · the coin owes nothing: 7 heads in 10, then 990 more flips outweigh them · insurer: thousands of homes → bill per home near $100 · one owner, one trader (Personal Finance 9): few tries
- **Pascal** (1623–1662) and **Fermat** (died 1665) · French mathematicians · letters of **1654** on a game stopped halfway ("problem of points") → start of probability · **Jacob Bernoulli** · Swiss (1654–1705) · *Ars Conjectandi* ("The Art of Conjecturing"), **1713** (after his death) → law of large numbers · **Daniel Bernoulli** · his nephew (1700–1782) · **1738**, St Petersburg game (named after the city's academy, which printed it): average prize with no limit, yet nobody pays much → the 100th dollar counts less (Microeconomics 11) · Kahneman and Tversky, 1979 (Behavioral 3)
- **Eleven tools to read any formula** · **1 · A line is a start and a slope: Q = a − bP** · **2 · Two equations, one answer: set them equal** · **3 · Growth multiplies; the rule of 70 counts the doublings** · **4 · Marginal = the slope of the total** · **5 · The top of the hill is where the slope is zero** · **6 · An area adds up the marginals** · **7 · Elasticity = % ÷ %, so the units cancel** · **8 · The best bundle is where the two slopes match** · **9 · Tomorrow is worth today ÷ (1 + r); a series adds up to 1 ÷ (1 − x)** · **10 · Expected value = each outcome × its chance, added up**

Watch out: do not **add the chances** (½ + ½ = 1 is not the flip's value) → **multiply** each prize by its own chance, then add · a loss is **−**, not + · E is an **average**, not a promise: a few tries can land far from it