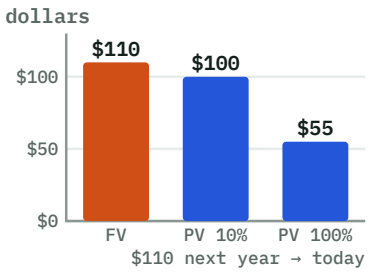
What is $110 next year worth today?

Growth **multiplies** by 1 + r · going back in time **divides** by it → **present value** · a stream where each round passes on a share x → **1 ÷ (1 − x)** · a payment forever = **payment ÷ r**. Tool 9 of the eleven.

1 Today's worth

Lia draws

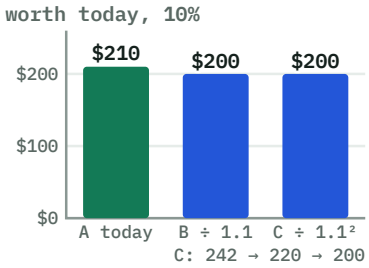
- **Lia** (draws) + **Sam** (formulas) · **Coralia** (toy island, dollars) · rate **r = 10%** a year = 0.1 · toys only: no real loan or investment, nothing is advice
- Forward (Personal Finance, Lesson 2): `$100 × 1.1 = $110` → `× 1.1 = $121`
- Backward: `$110 ÷ 1.1 = $100` · $110 next year = **$100 today** (not $110, not "about $90")
- Higher rate → today's bar **smaller** · $110 next year: r 0% → $110 · 10% → **$100** · 100% → $55
- **Present value** (PV) = what money promised for later is worth today



2 The formula

Sam writes · PV = FV ÷ $(1 + r)^t$

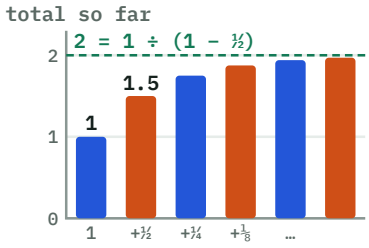
- **FV** = future value (amount promised) · **PV** = present value · **r** = rate as a decimal · **t** = years · $(1 + r)^t$ = 1 + r times itself t times (Math Lesson 3)
- One ÷ (1 + r) per year back · `$121 ÷ 1.1 ÷ 1.1 = $100` = `$121 ÷ 1.21`
- **Discount factor** `1 ÷ (1 + r)` ≈ 0.91 at 10% · `$220 ÷ 1.1 = $200` (not $198: the 10% is of today's amount)
- Practice: $120 next year at 20% → **$100** · at 50% → $80 · $144 in 2 years at 20% → $120 → **$100**
- **Compare on one date**: A $210 today · B $220 next year → $200 · C $242 in 2 years → $200 → **A** worth most



3 A stream forever

the series

- `1 + x + x² + x³ + … = 1 ÷ (1 − x)` for a share x from 0 to below 1 · "…" = forever · **geometric series** (university name)
- x = ½: 1, 1.5, 1.75, 1.875 … → **2**, never past · each term closes half the gap left
- Why, one line: S = 1 + x·S → S − x·S = 1 → `S = 1 ÷ (1 − x)`
- **Perpetuity** = a payment every year forever, first one next year · x = 1 ÷ 1.1 · list starts at x: `x + x² + … = x ÷ (1 − x) = 10` → **payment ÷ r** · $10 ÷ 0.1 = **$100** · $30 ÷ 0.1 = $300 ($330 = an extra $30 today)
- Rate ↑ → worth ↓ · field, $12 rent forever: 10% → **$120** · 20% → $60 · 40% → $30 · rate × 2 → worth ÷ 2



4 Three courses, one series

x = share passed on

Where	x = share to the next round	1 ÷ (1 − x)
Money multiplier · Macroeconomics 6	banks keep 10% (reserve ratio), lend 90% → x = 0.9 · practice: lend 80% → 0.8	1 ÷ 0.1 = **10** · 1 ÷ 0.2 = **5**
Spending multiplier · Macroeconomics 5	MPC (marginal propensity to consume) = share of each extra $ spent = 0.8	1 ÷ 0.2 = **5** · $1 + $0.8 + $0.64 + … = $5
Cooperating forever · Game Theory 5	δ (delta) = how much tomorrow counts = 0.9	1 ÷ 0.1 = **10**
Perpetuity · this lesson	x = 1 ÷ (1 + r) = 1 ÷ 1.1	× payment, minus the first 1 → payment ÷ r

- Not the series: rule of 70 (doublings, Math Lesson 3) · ½ × base × height (area, Math Lesson 6) · tax $ × quantity (rectangle)

5 So what?

tool 9 of 11

- **Irving Fisher** · American economist at Yale (1867–1947) · *The Rate of Interest* (**1907**), *The Theory of Interest* (**1930**): interest = the price of waiting · **Richard Kahn** · English economist at Cambridge · the multiplier (*Economic Journal*, June **1931**) · credited by **John Maynard Keynes** (English, 1883–1946) in *The General Theory* (**1936**, ch. 10) · exams: AP Macroeconomics (US high-school exam) → both multipliers · check your board's syllabus
- **Eleven tools to read any formula** · **1 · A line is a start and a slope: Q = a − bP** · **2 · Two equations, one answer: set them equal** · **3 · Growth multiplies; the rule of 70 counts the doublings** · **4 · Marginal = the slope of the total** · **5 · The top of the hill is where the slope is zero** · **6 · An area adds up the marginals** · **7 · Elasticity = % ÷ %, so the units cancel** · **8 · The best bundle is where the two slopes match** · **9 · Tomorrow is worth today ÷ (1 + r); a series adds up to 1 ÷ (1 − x)**

Watch out: never add dollars of **different dates**: $100 today + $110 next year at 10% = **$200** today, not $210 → bring every amount to **one date** first · `$220 ÷ 1.1 = $200`, not 220 − 10% = $198 · the series needs a share **x below 1** (x = 1 or more → no total) · `x + x² + …` (no 1) = x ÷ (1 − x)