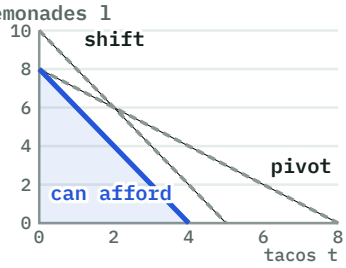
# How do you choose with a budget?

Budget line = every bundle that spends all the money · curve of equal points = every bundle worth the same · best bundle → the curve **just touches** the line → **the two slopes match**. Tool 8 of the eleven.

## 1 The budget line

Lia draws

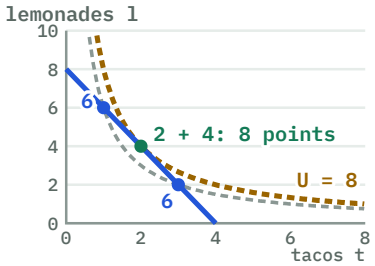
- **Lia** (draws) + **Sam** (formulas) · **Coralia** (toy island, dollars) · Lia: **$16**, taco **$4**, lemonade **$2** (Microeconomics, Lesson 11 had $12)
- `4t + 2l = 16` · t = tacos, l = lemonades · **bundle** = one mix (t, l) · ends: 16 ÷ 4 = **4 tacos**, 16 ÷ 2 = **8 lemonades**
- Under the line → affordable, money left · over → cannot pay
- **Slope** = `−(4 ÷ 2) = −2`: +1 taco → −2 lemonades = opportunity cost (Microeconomics, Lesson 1; slope, Math Lesson 1)
- Dashed: new **price** → line **pivots** (one end moves): taco $2 → end 8, slope −1 · new **budget** → line **shifts**, same slope: $20 → 5 and 10




## 2 Equal-points curves

indifference curves

- Toy taste: **points** `U = t × l` (U = utility; "I like a mix") · a **function of two variables**: a pair (t, l) → one number
- **Indifference curve** = all bundles worth the same points · higher curve = better
- On the line: 1 + 6 → **6** · 2 + 4 → **8** · 3 + 2 → **6** · corners 4 + 0, 0 + 8 → 0
- Curve **crosses** the line → a better bundle exists · curve **just touches** → best: **2 tacos + 4 lemonades** · grey dashed: U = 6, crosses at 1 + 6 and 3 + 2
- **Marginal rate of substitution** (MRS) = curve slope `−l ÷ t`: lemonades Lia would give up for a little more taco · −6 at 1 + 6 · −2 at 2 + 4 · −⅔ at 3 + 2




## 3 One equation

Sam writes

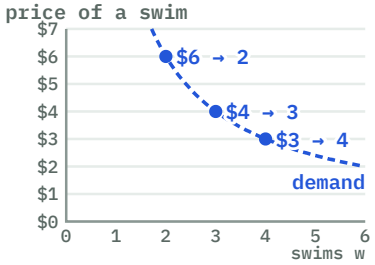
- Line into the points: `l = 8 − 2t` → `U = t(8 − 2t) = 8t − 2t²` · `U′` ("U prime") = its slope
- Top of the hill (Math Lesson 5): `U′ = 8 − 4t = 0` → **t = 2**, l = 8 − 4 = **4**, 8 points · check 6, 8, 6 → a top (one top only; second derivative at university)
- Practice: pool $12, slide $2, ice pop $1 → `U = 12s − 2s²`, `U′ = 12 − 4s` → 3 + 6, 18 points
- Fair $20, ride $5, drink $1 → `U = 20r − 5r²` · slope of 5r² = **10r** (not 5r) → r = 2, d = 10, 20 points

| t | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| l = 8 − 2t | 8 | 6 | 4 | 2 | 0 |
| U = t × l | 0 | 6 | 8 | 6 | 0 |

## 4 Slopes match

why it works

- Best bundle ⇔ `curve slope = line slope` · `−l ÷ t = −(P_taco ÷ P_lemonade)` · 4 ÷ 2 = 2 ✓
- Curve **steeper** (1 + 6: −6 vs −2) → a taco is worth 6 lemonades, costs 2 → **buy more tacos** · flatter → fewer
- Same as the **last-dollar rule** (Microeconomics, Lesson 11): points per $ · 1 + 6: taco 1.5, lemonade 0.5 · 2 + 4: **1 and 1**
- With points = `one good × the other`: slopes match ⇒ **same $ on each good** · Lia $24, snack $2, swim $6 / $4 / $3 → $12 each → swims **2, 3, 4** → **demand curve**
- Practice: Sam $18, pizza $3, cookie $1 → **3 + 9**, 27 points, curve −9 ÷ 3 = −3 = line −3




## 5 So what?

tool 8 of 11

- **Francis Edgeworth** · Irish economist (1845–1926) · *Mathematical Psychics* (**1881**): first indifference curves · **Vilfredo Pareto** · Italian (1848–1923) · *Manuale di economia politica* (**1906**): choice from the curves, credited to Edgeworth · **Joseph-Louis Lagrange** · born in Turin (1736–1813) · *Mécanique analytique* (**1788**): multiplier λ (lambda), the university way → same 2 + 4 · exams: budget lines + indifference curves in some A-level boards (UK school exams; Cambridge International 9708) → check your syllabus
- **Eleven tools to read any formula** · **1 · A line is a start and a slope: Q = a − bP** · **2 · Two equations, one answer: set them equal** · **3 · Growth multiplies; the rule of 70 counts the doublings** · **4 · Marginal = the slope of the total** · **5 · The top of the hill is where the slope is zero** · **6 · An area adds up the marginals** · **7 · Elasticity = % ÷ %, so the units cancel** · **8 · The best bundle is where the two slopes match**

**Watch out:** a **corner** (4 + 0 or 0 + 8) scores `4 × 0 = 0` · a slope slip sends you there (`U′ = 20 − 5r` × → r = 4, d = 0) · price → **pivot**, budget → **shift** · a new taste moves the **curves**, not the line · `U = t × l` is made up: new taste → new curves, same rule