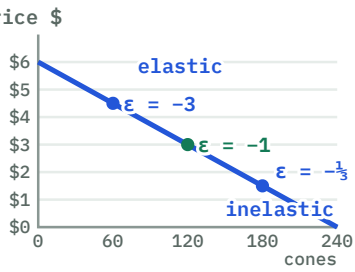
Why does elasticity need a percentage?

Slope = units per unit (cones per $) → changes with the units · elasticity `ε = %ΔQ ÷ %ΔP` → no unit, comparable everywhere. One straight line: same slope, different ε at every point. Tool 7 of the eleven.

1 Why not the slope?

Lia draws

- **Lia** (draws) + **Sam** (formulas) · **Coralia** (toy island, dollars) · the kiosk (Microeconomics, Lesson 2): demand `Q = 240 − 40P` · slope **−40** cones per $ everywhere (Lesson 1)
- Slope has a **unit**: −40 cones per $ = **−0.4 cones per cent** = other number in crates of 12 → can't compare goods, countries, charts
- **Elasticity of demand** `ε = %ΔQ ÷ %ΔP` · cones ÷ cones, $ ÷ $ → **units cancel** · ε (epsilon, Greek e) · %Δ = "the % change in"
- Demand: ε **negative** (price ↑ → cones ↓) · size = `|ε|` (minus sign dropped)
- "Stretchy" = small price change → big change in Q, **in proportion**



2 ε at a point

Sam writes

- 1% price rise = P ÷ 100 dollars → Q moves slope × P ÷ 100 → **point elasticity** `ε = slope × P ÷ Q` (slope = derivative, Lesson 4)
- Kiosk: `$4.50, 60: −40 × 4.5 ÷ 60 = −3` · `$3, 120: −1` · `$1.50, 180: −⅓`
- Straight line: **top half elastic** · **middle = −1** · **bottom half inelastic**

ELASTIC `|ε| > 1`	UNIT `|ε| = 1`	INELASTIC `|ε| < 1`
1% dearer → more than 1% fewer · $4.50 (3), $5 (5) · bus +10% → −30%	exactly 1% fewer · $3 at the kiosk · $4.50 on lemonade `180 − 20P`	less than 1% fewer · $1.50 (⅓), $2 (½), $1 (⅕) · salt +20% → −4%

3 A step: midpoint

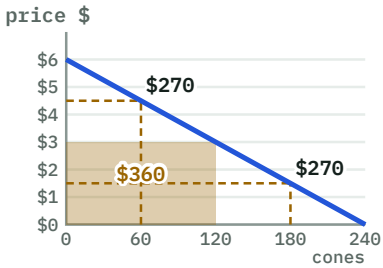
two points, no slope

- From an end → 2 answers: $2 → $4 (160 → 80) `−50% ÷ +100% = −½` · back down `+100% ÷ −50% = −2` ×
- **Midpoint formula**: each change ÷ **average of the two ends** → up = down
- `%ΔQ = −80 ÷ 120 = −⅔` · `%ΔP = 2 ÷ 3 = ⅔` · `ε = −1` = point ε at $3 (middle of the step)
- Practice: museum $4 → $6, 140 → 60 · averages $5, 100 · `−80% ÷ 40% = −2` · elastic
- Many textbooks (Gregory Mankiw, American economist, Harvard) + courses use it · boards differ → check your syllabus (IB HL = International Baccalaureate higher level · A-level = UK · AP = US)

4 Revenue test

a hill, Lesson 5

- Total revenue `TR = P × Q` = rectangle · kiosk `$1.50: 270` · `$3: 360` · `$4.50: 270`
- Elastic → price **cut** ↑ revenue · inelastic → cut ↓ revenue · top of the hill (slope 0) ⇔ `|ε| = 1`
- Practice: lemonade top at $4.50 (`4.5 × 90 = $405`) · museum rise $560 → $360 · juice cut $4 → $3: $400 → $450
- Supply too: `ε = slope × P ÷ Q`, **positive** · kiosk `Q = 60P − 60`: $2 → 2, $3 → 1.5 · line from 0 (`Q = 40P`) → 1 everywhere



5 So what?

tool 7 of 11

- **Alfred Marshall** · English economist, Cambridge · *Principles of Economics* (**1890**), Book III, ch. IV · made "elasticity of demand" a standard tool · defined by % (1% price fall → % rise in amount bought) · elastic / inelastic
- **Eleven tools to read any formula** · **1 · A line is a start and a slope: Q = a − bP** · **2 · Two equations, one answer: set them equal** · **3 · Growth multiplies; the rule of 70 counts the doublings** · **4 · Marginal = the slope of the total** · **5 · The top of the hill is where the slope is zero** · **6 · An area adds up the marginals** · **7 · Elasticity = % ÷ %, so the units cancel**

Watch out: slope ≠ elasticity · juice bar `Q = 300 − 50P` at $4: `ε = −50` × → `−50 × 4 ÷ 100 = −2` ✓ · steep ≠ inelastic everywhere · compare `|ε|` with 1, not ε