How big is the triangle?

An area on a price chart = dollars (base in units × height in $) → triangle `½ × base × height`, rectangle `base × height`. Surplus, tax revenue, deadweight loss, variable cost = areas. ∫ = "add up" (the integral). Tool 6 of the eleven.

1 Why a triangle is money

Lia draws

- **Lia** (draws) + **Sam** (formulas) · **Coralia** (toy island, dollars) · the kiosk (Microeconomics, Lesson 2): demand `Q = 240 − 40P` (starts at $6) · supply `Q = 60P − 60` (starts at $1) → **$3, 120 cones**
- Each buyer: would pay up to their point on demand − pays $3 = **their gain** ($5 − $3 = $2) → all the gaps added = the **triangle**
- **Consumer surplus** = what buyers would have paid − what they paid · `½ × 120 × (6 − 3) = $180`
- **Producer surplus** = what sellers got − the least they would accept · `½ × 120 × (3 − 1) = $120` · total gain from trade **$300**
- Lower price → more buyers + bigger gaps → triangle grows · gain at price p: `20(6 − p)²`



2 ½ × base × height

Sam writes

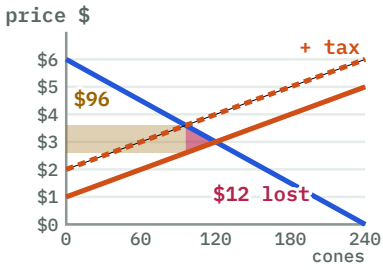
TRIANGLE ½ × b × h	RECTANGLE b × h	NOT AN AREA
slanted line from 0 → height grows · consumer + producer surplus · deadweight loss · area under a rising MC	one height all along · tax collected · money taken in (P × Q) · area under a flat MC ($5 × 30 = $150)	one point or one height · a price · one MC (4 × 10 = $40) · the crossing

- **Units**: base in cones × height in $ = **$** · height = from the price up to where demand starts (or down to where supply starts)
- Practice: smoothies `160 − 20P = 40P − 80` → **$4, 80** · buyers `½ × 80 × 4 = $160` · stand `½ × 80 × 2 = $80`

3 A tax

rectangle + triangle

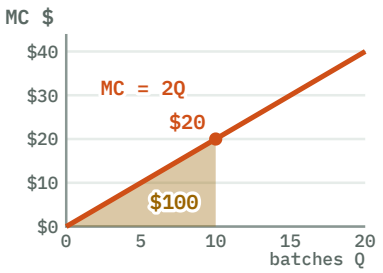
- Tax = a **wedge** (Lesson 2): $1 → buyers pay **$3.60**, kiosk keeps **$2.60**, **96** cones
- **Tax revenue** = tax × units sold **with** the tax = rectangle · `1 × 96 = $96`
- **Deadweight loss** = gain of trades that no longer happen · nobody gets it · `½ × 1 × (120 − 96) = $12`
- Tax × 2 → loss × 4 (straight lines): $2 → 72 cones · `½ × 2 × 48 = $48`
- Practice: taxis, $3 tax → 40 rides at $8 / $5 · collected `3 × 40 = $120` · lost `½ × 3 × 20 = $30` · riders `(8 − 6) × 40 = $80`, drivers $40 · check: gains lost = collected + lost (bread: 70 = 60 + 10)



4 Area = sum of marginals

the integral

- Cookie stand (Lesson 4): marginal cost (extra cost of 1 more batch) `MC = 2Q` = slope of total cost · add the extra cost of every little bit → **area under MC**
- 0 to 10 batches: `½ × 10 × 20 = $100` = **variable cost** · total cost TC(10) = $100 fixed (paid at 0 batches) + $100 = **$200**
- ∫ = stretched S for "sum" · `∫ MC dQ` = add up MC over every tiny step dQ = **integral** (university name, never needed)
- Between two quantities = difference: `MC = 4Q`, vases 6 to 10 = `200 − 50 = $150` = `½ × (20 + 40) × 5`



5 So what?

tool 6 of 11

- **Gottfried Leibniz** · German mathematician and philosopher · unpublished manuscript of **29 October 1675** · first ∫, a long S for Latin *summa* ("sum"), in place of "omn." · in print **1686** · Exams: IB Economics HL (International Baccalaureate, higher level) asks surplus + tax areas from straight lines
- **Eleven tools to read any formula** · **1 · A line is a start and a slope: Q = a − bP** · **2 · Two equations, one answer: set them equal** · **3 · Growth multiplies; the rule of 70 counts the doublings** · **4 · Marginal = the slope of the total** · **5 · The top of the hill is where the slope is zero** · **6 · An area adds up the marginals**

Watch out: forgetting the **half** · `120 × 3 = 360` × → `½ × 120 × 3 = $180` ✓ · and the rectangle's base = units sold **with** the tax (bread: 2 × 30, not 2 × 40)