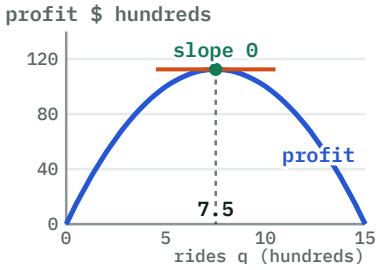
Where is the top of the hill?

The top of a hill = where the slope is zero → take the slope, set it to 0, solve (first-order condition). For profit: slope = MR – MC → the top reads MR = MC. Price from demand. Tool 5 of the eleven.

1 The profit hill

Lia draws · Sam writes

- **Lia** (draws) + **Sam** (formulas) · **Coralia** (toy island, dollars) · the only ride company (Microeconomics, L7) · demand `P = 40 − 0.02Q`
- Count in **hundreds**: `q = Q ÷ 100` → `P = 40 − 2q` · q = 7.5 = 750 rides · money in $ hundreds
- **Revenue** `TR = P × q = 40q − 2q²` · **cost** `10q` (marginal cost MC = $10 a ride) · **profit** = TR – cost = `30q − 2q²` · fixed cost $5,250/day (paid at 0 rides) → slope 0 → hill lower, top unchanged → left out
- Profit: **0, 100, 100, 0** at q = 0, 5, 10, 15 → rises, top between 5 and 10, falls
- **Tangent** (Lesson 4) = line touching at one point · slope + left of the top · **0 at the top** · – right of it



2 Slope = 0

first-order condition

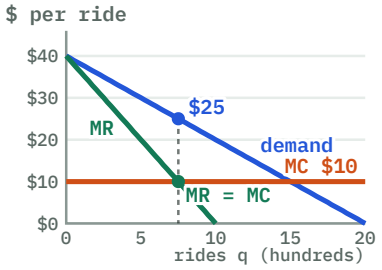
1 · the hill	2 · its slope (power rule)	3 · top: slope = 0	4 · solve
`30q − 2q²`	`30 − 4q` ($2q^2 \to 4q$)	`30 − 4q = 0`	**q = 7.5** (750 rides) · P = 40 – 15 = **$25**

- **First-order condition** = set the slope of what you maximize to 0 (university name, never needed) · $20q - q^2 \to 20 - 2q = 0 \to$ **q = 10**
- Practice: bike shop `P = 50 − 2q`, $10 a rental → profit `40q − 2q²` → 40 – 4q = 0 → **q = 10** · P = **$30** · profit 200 (198 at 9 and 11) · at q = 8: slope +8 → still rising

3 MR = MC

the same sentence

- **Marginal revenue** = extra revenue from one more = slope of TR · `MR = 40 − 4q` · **MC** = extra cost of one more = slope of cost = $10
- slope of profit = `MR − MC` = 0 ⟺ **MR = MC** · 40 – 4q = 10 → q = 7.5 (same top)
- MR **twice as steep** as demand: $-2q^2 \to -4q$ (Microeconomics L7's rule, with its reason)
- **Price from demand**, not MR: 40 – 2 × 7.5 = $25 · MR > MC → sell more · MR < MC → sell less
- Practice: hall `60 − 3q`, MC $12 → MR `60 − 6q` = 12 → **q = 8**, P = **$36** · ferry `36 − 2q`, MC $8: MR = 36 – 2q is the slip (q 14, P $8 = MC, profit 0) → MR `36 − 4q` → **q = 7, $22**, profit 98



4 Top or not?

check the sides

A MAXIMUM	NOT A MAXIMUM
MR = MC (one hill) · flat tangent on a hill · profit **112 · 112.5 · 112** at q = 7 · 7.5 · 8	revenue's top (q = 10, MR = 0: profit 100 < 112.5) · steepest point · **valley bottom** (flat, but a minimum) · highest profit per ride (the first ride)

- One-top hill only · at university: the **second derivative** (slope of the slope) · **firm among many** (Microeconomics L6) = price taker → MR = P → top: **P = MC** · strawberries $20, `TC = 50 + Q²` → 20 – 2Q = 0 → **Q = 10**, profit $50 · at $30 → Q = 15 (MC = supply)

5 So what?

tool 5 of 11

- **Pierre de Fermat** · French lawyer and mathematician · **about 1636** · highest/lowest value where a quantity stops changing · **Antoine Augustin Cournot** · French (1801–1877) · book of **1838** · demand as a function, slope = 0 for a monopolist's best output · among the first calculus in economics
- **Eleven tools to read any formula** · **1 · A line is a start and a slope: Q = a – bP** · **2 · Two equations, one answer: set them equal** · **3 · Growth multiplies; the rule of 70 counts the doublings** · **4 · Marginal = the slope of the total** · **5 · The top of the hill is where the slope is zero**

Watch out: stopping where **revenue** is highest (MR = 0, q = 10) · cost still rises → profit 100, not 112.5 · and the price comes from **demand**, never from MR