What does “marginal” really mean?

Marginal = the slope of the total at one point → the tangent → the derivative. Power rule: Q^2 → 2Q. Average = total ÷ units; a marginal below the average pulls it down. Tool 4 of the eleven.

1 Step or point

Lia draws · Sam writes

- **Lia** (draws) + **Sam** (formulas) · **Coralia** (toy island, dollars) · cookie stand: `TC = 100 + Q²` · $100 oven + Q^2 ingredients and time (Q = batches, `Q² = Q × Q`)
- TC: **$100, $125, $200, $221, $500** at Q = 0, 5, 10, 11, 20 · curve bends up
- **Marginal cost** (Micro 5) = extra cost of one more · `MC = ΔTC ÷ ΔQ` · **Δ** = “the change in” · 10 → 11: 221 − 200 = **$21**
- **Tangent** = straight line that just touches the curve at one point · its slope = the curve's slope there · at Q = 10: **$20** (5 → $10 · 20 → $40)
- Shrink the step: 10, 5, 2, 1 batches → $30, $25, $22, $21 → **$20** · **derivative** = the slope at one point · `TC'` (“TC prime”) or `dTC/dQ` = MC



2 Power rule

a rule, not a ruler

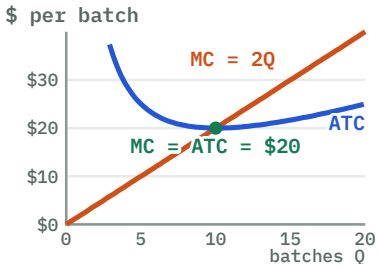
total	Q^2	Q^3	5Q (aQ)	100 (a plain number)
its slope	2Q	$3Q^2$	5 (a)	0 · fixed cost ⇒ no effect on MC

- **Power** = small raised number · comes down in front → goes **one lower** · a number in front stays: `3Q² → 3 × 2Q = 6Q`
- `TC = 100 + Q²` ⇒ `MC = 0 + 2Q = 2Q` ⇒ Q = 10: **$20** = the tangent
- Practice: `72 + 2Q²` → **4Q** = $24 at Q = 6 (step 6 → 7: $26) · `50 + 4Q + Q³` → `4 + 3Q²` = **$16** at Q = 2 (not 3Q → $10) · check: slope at a point sits **between** the steps on each side ($11 · $23)

3 Marginal vs average

slope vs ratio

- **Average total cost** `ATC = TC ÷ Q = 100 ÷ Q + Q` · $25 at 5 · **$20** at 10 · $25 at 20
- MC **below** ATC ⇒ ATC **falls** (a low score pulls an average down) · MC **above** ⇒ ATC **rises**
- MC = ATC at the **lowest** average: Q = 10, **$20** (Micro 5's rule, with its reason)
- Practice: print shop `100 + 10Q + Q²` → `MC = 10 + 2Q` · Q = 10: MC $30 = ATC 300 ÷ 10 · Q = 5: MC $20 < ATC $35 → falling



4 Sort it

marginal · average · total

MARGINAL · ONE MORE · A SLOPE	AVERAGE · PER UNIT · A RATIO	TOTAL · THE WHOLE AMOUNT
cost of one more batch · extra money from one more ticket · a car's speed now (80 km/h) · fuel for the next km · tax on the next dollar	cost per batch · income per person · 120 km ÷ 2 h = 60 km/h · fuel per km · points per game	the day's whole bill · 120 km · fuel for the whole trip · points in a season

- **A marginal that falls** (practice): orchard `Q = 30L − L²` baskets, L pickers → **marginal product** `MP = 30 − 2L` (the minus stays) · L = 10: MP **10** < average 200 ÷ 10 = **20** → average falls · 11th picker adds 9 (a step) · MP = 0 at 15 → top of the hill (Lesson 5)

5 So what?

tool 4 of 11

- **Isaac Newton** · English (1643–1727) · slope at a point (start of calculus), “fluxions”, **1665–66** (Cambridge closed for the plague) · **Gottfried Leibniz** · German (1646–1716) · same idea on his own, printed first, **1684** · the *d* of dTC/dQ
- Quarrel over who was first → Royal Society (England's academy of science) sides with Newton, **1712** · today: both credited · the apple = a legend about gravity, told by Newton late in life
- “Marginal” in economics ← **Jevons, Menger, Walras** (English, Austrian, French), **1871–74** · spread by **Alfred Marshall** (English) in his textbook, 1890 (Thinkers 4) → marginal cost, marginal revenue, marginal utility
- **Eleven tools to read any formula** · **1 · A line is a start and a slope: Q = a − bP** · **2 · Two equations, one answer: set them equal** · **3 · Growth multiplies; the rule of 70 counts the doublings** · **4 · Marginal = the slope of the total**

Watch out: reading the marginal as **total ÷ Q** = that is the **average** · at 20 batches 500 ÷ 20 = $25, but one more costs about **$40** (2 × 20) · and a step (10 → 11: $21) ≠ a point ($20)