How fast is it growing?

Adding → the same amount each step · growing → the same factor (number multiplied by, e.g. × 1.07) each step. 7% a year for 10 years = about double, not 70% more. Rule of 70 counts the doublings · the log undoes a power. Tool 3 of the eleven.

1 Adds or multiplies

Lia draws · Sam writes

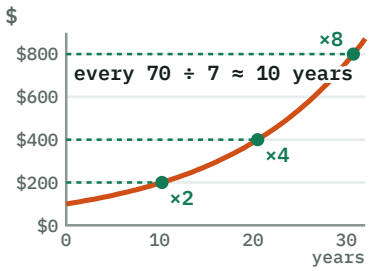
- **Lia** (draws) + **Sam** (formulas) · **Coralia** (toy island, dollars) · $100 for 10 years
- **Adds** $7 a year → `A = 100 + 7t` → **$170** · straight line
- **Grows** 7% a year → × `1.07` (100% + 7%) → `A = 100 × 1.07^t` → **$197** · curve bends up
- **Exponent** = small raised number · `1.07³ = 1.07 × 1.07 × 1.07` · **exponential growth** = × same number again and again ⇒ the amount added keeps growing
- Year 20: $240 vs **$387** · by eye: $170 / $200 / $300?



2 Rule of 70

count the doublings

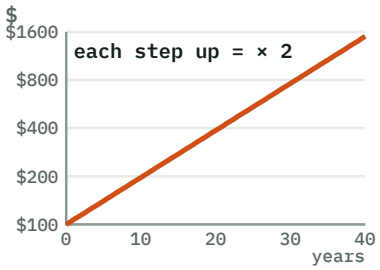
- **Years to double ≈ 70 ÷ rate (%)** · 7% → **10** · 3.5% → **20** · 5% → 14 · 2% → 35 · Macro 10: 2.3% → about 30
- Turned round: **rate ≈ 70 ÷ years** · prices doubled in 7 years → about **10%** a year
- **Doubling time** = same wherever you start · two doublings = **× 4**: `$100 → $200 → $400` (not $300)
- Finance 2: rule of **72** = same rule, divides evenly by 6, 8, 9, 12
- Practice: $200 at 5% → ≈ $400 at year **14**, ≈ $800 at 28 · Lia +$10 a year → $340, $480 · prices +2% → $2 loaf → **$8** after 70 years



3 The log

undo a power

- **Logarithm (log)** = how many times we multiplied · `10³ = 1,000` ⇒ `log 1,000 = 3`
- Turns × into + : `log(100 × 1,000) = 2 + 3 = 5`
- **Log scale** (Data 1): each step up = same **multiple** · $100, $200, $400, $800, $1,600 ⇒ steady growth = **straight line** · linear scale: same **amount** ⇒ it bends
- **e** ≈ `2.718` · interest paid every instant · **ln** = log in base e (university formulas) · `ln 2 ≈ 0.69` ⇒ 69 ÷ rate → rounded to **70**



4 Sort it

add or multiply?

ADDS (LINEAR) · + SAME AMOUNT	MULTIPLIES (EXPONENTIAL) · × SAME FACTOR
pay +$1,000 a year · tank +5 liters a minute · a bus every 10 minutes · rent +$50 a year · taxi +$2 a km · plant +2 cm a week	population +2% · balance at 4% · a loaf doubling weekly (History 4) · prices +3% · bacteria ×2 an hour · unpaid debt +2% a month

- **Shrinking** too: prices +7% a year ⇒ $100 buys about **$50** after 10 years, **$25** after 20 (not 100 − 70% = $30) · falls fast, then slower, never $0
- **%** vs **percentage points** (Data 3): 3% → 5% = +2 points, not +2% · 7% → 3.5% = −3.5 points = a 50% fall

5 So what?

tool 3 of 11

- **John Napier** · Scottish mathematician (1550–1617) · first log tables, **1614** · multiply → add
- **Jacob Bernoulli** · Swiss · **1683**: $1 at 100% paid more and more often → stays between $2 and $3 → first glimpse of e · **Leonhard Euler** · Swiss · the letter e (1731), its value (**1748**)
- Growth of countries, prices, savings → Macroeconomics, Personal Finance, Economic History
- **Eleven tools to read any formula** · **1 · A line is a start and a slope: Q = a − bP** · **2 · Two equations, one answer: set them equal** · **3 · Growth multiplies; the rule of 70 counts the doublings**

Watch out: adding the rates over the years · 7% × 10 ≠ 70% more → **97%** · 3 years of 10%: `1,000 → 1,100 → 1,210 → 1,331` = **33.1%**, not 30% · each year's gain must grow (+100, +110, +121)