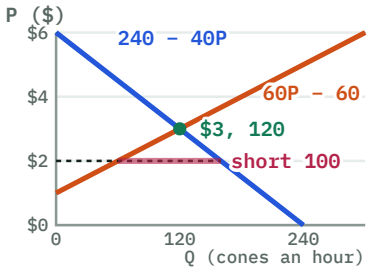
Where do two lines cross?

Two lines → one crossing → one price and one quantity. Set the formulas equal, do every step to both sides, check in both. The kiosk at $3 and 120, a shift, a tax, a subsidy. Tool 2 of the eleven.

1 The crossing

Lia draws · by eye

- **Lia** (draws) + **Sam** (formulas) · **Coralia** (toy island, dollars) · the beach kiosk of Micro Lesson 2 · P = price ($), Q = cones an hour
- **Demand** `Q = 240 − 40P` (cones wanted) · **supply** `Q = 60P − 60` (cones made) · Math Lesson 1
- Crossing = cones wanted **=** cones made ⇒ the **market price** · by eye ≈ $3 · near is not an answer
- Below it → **shortage** (wanted > made) → price ↑ · `$2: 160 − 60 = 100` short · above it → **surplus** (made > wanted) → price ↓



2 Set them equal

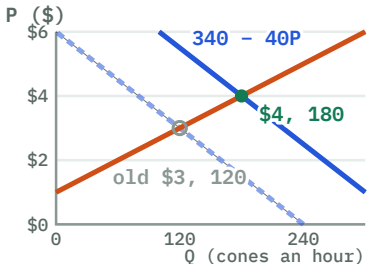
Sam writes · 3 steps

- **Golden rule:** the same step to **both** sides (add, take away, multiply, divide)
- `240 − 40P = 60P − 60` → `+40P: 240 = 100P − 60` → `+60: 300 = 100P` → `÷100: P = 3`
- Q: put P into either formula → `240 − 40 × 3 = 120` · **check** in the other → `60 × 3 − 60 = 120` ✓ (if not: a slip)
- **System of equations** = two equations true at once · the crossing = the one point that fits both
- Practice · lemonade `180 − 20P = 40P − 60` → `240 = 60P` → **$4, 100 cups** · juice bar `200 − 30P = 20P − 50` → `250 = 50P` → **$5, 50**

3 A shift

a new start

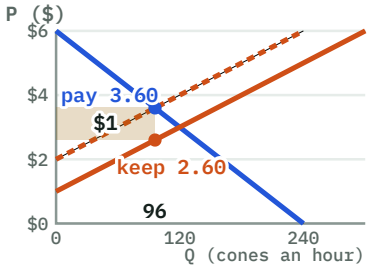
- Festival: demand `Q = 340 − 40P` · start (Q at $0) 240 → 340 · slope (cones per $1) still −40
- `340 − 40P = 60P − 60 → 400 = 100P → $4` · `Q = 340 − 160 = 180` = `60 × 4 − 60` ✓
- More demand ⇒ crossing slides **up the supply line** · bikes `200 − 25P` vs `25P − 50`: $5, 75 → summer `300 − 25P`: $7, 125



4 A wedge

tax · subsidy

- **Tax** $1 a cone (Micro Lesson 4) = a **wedge** · buyers pay **Pb** (b = buyers), kiosk keeps `Pb − 1` → `240 − 40Pb = 60(Pb − 1) − 60`
- open the parentheses: `60Pb − 60 − 60 = 60Pb − 120` → `360 = 100Pb` → buyers **$3.60** · kiosk **$2.60** · `Q = 240 − 144 = 96` · shared: +60¢ / −40¢
- **Subsidy** = paid to the seller per unit → keeps `Pb + s` (s = subsidy a unit) · lemonade $1.50: `40(Pb + 1.50) − 60 = 40Pb` → `180 = 60Pb` → buyers **$3** · stand **$4.50** · 120 cups (was $4, 100)



5 Which way?

4 moves

DEMAND START ↑	SUPPLY START ↓	SUPPLY START ↑	DEMAND START ↓
price ↑ · Q ↑ · up the supply line · heat wave `300 − 40P`: $3.60, 156	price ↑ · Q ↓ · up the demand line · 50¢ tax `60P − 90`: $3.30, 108	price ↓ · Q ↑ · down the demand line · cheaper milk `60P`: $2.40, 144	price ↓ · Q ↓ · down the supply line · rainy day `200 − 40P`: $2.60, 96

- Kiosk's own price change = a move **along** a line, nothing shifts

6 So what?

tool 2 of 11

- **Al-Khwarizmi** · scholar of Persian origin in Baghdad · book on al-jabr (“completion”, the 820s) → move a term across the = sign → “algebra”
- **Léon Walras** · French economist · Éléments d'économie politique pure (Elements of Pure Economics), **1874** → a whole economy as equations solved together · the name of this site
- **Eleven tools to read any formula** · **1 · A line is a start and a slope: Q = a − bP** · **2 · Two equations, one answer: set them equal**

Watch out: changing **one side only** · `200 = 50P − 50`: remove −50 by **adding** 50 to both → `250 = 50P` → $5, not `150 = 50P` → $3 · the check catches it: at $3, 110 wanted vs 10 made ×