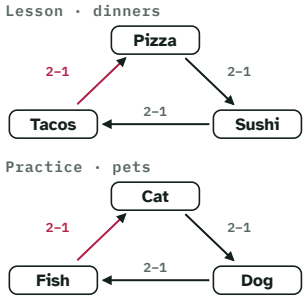
Can voting be fair?

Voting rule = a way to turn everyone's rankings into one winner · majority rule can go **round in a circle** → the **agenda** picks the winner · views on one line → the **median voter** wins · same ballots, different rules → different winners · **Arrow**: no rule is perfect.

1 Majority rule

and the cycle

- **Your move**: pick the dinner you want to win (never graded or stored)
- **Majority rule**: two options at a time → the one more voters prefer wins
- Ana Pizza > Sushi > Tacos · Ben Sushi > Tacos > Pizza · Cam Tacos > Pizza > Sushi → pizza beats sushi, sushi beats tacos, tacos beat pizza (each `2–1`)
- **Condorcet cycle**: "the group prefers A to B, B to C and C to A" → no option beats both others · **Marquis de Condorcet**, French mathematician, **1785** · each ranking sensible, the group's wishes go in a circle
- Practice · pets: Ana Cat > Dog > Fish · Ben Dog > Fish > Cat · Cam Fish > Cat > Dog → Cat beats Dog, Dog beats Fish, Fish beats Cat



2 The agenda

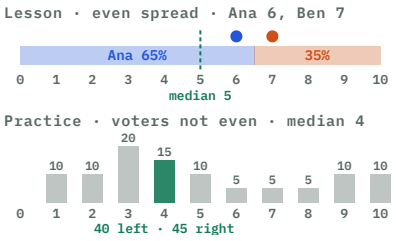
who sets the order

- **Agenda**: "the order in which options are put to the vote" · two first → winner against the third
- Cycle ⇒ the order picks the winner ⇒ **whoever sets the agenda has power** · any of the three can win
- Want tacos? → pizza vs sushi first → pizza → loses to tacos (tacos never face sushi)
- Practice · want Dog? → **Fish vs Cat** first → Fish → Dog beats Fish (Dog never faces Cat)
- Chain: rankings point different ways → majority goes round in a circle → order of votes picks the winner → agenda setter has power

3 Median voter

views on one line

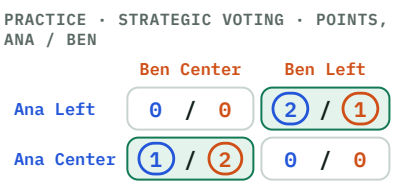
- Voters on one line 0 (far left) → 10 (far right) · each votes for the **nearer** candidate · split at the halfway point (beach of Lesson 2)
- Even spread · Ana 3, Ben 7 → halfway 5 → **tie** · Ben at 7 → Ana's best spot **6** → `65%` (5 → 60%, on top of Ben → 50%)
- They move in turns → both at **5** = **Nash equilibrium** (nobody gains by moving alone; John Nash, 1950)
- **Median voter**: the middle of the crowd, fewer than half of the voters on each side (even spread → the middle, 5) · one line ⇒ majority rule always has a winner: the median's favorite ⇒ platforms (promises) look alike
- Practice · voters not even → running total passes 50 at **4** → median 4, not 5 · Ana 4 vs Ben 5 → `55–45` → both end at 4



4 Same ballots

different rules

- Race · 40 Right > Center > Left · 35 Left > Center > Right · 25 Center > Left > Right
- **Plurality** (most first choices, even without a majority = more than half) → **Right** 40 · **Runoff** (top two again; **ranked-choice**: drop the last, pass ballots on) → **Left** `60–40` · one on one → **Center** `65–35`, `60–40`
- **Condorcet winner**: beats every rival one on one · **Approval**: tick as many as you like, most ticks wins
- Burlington, Vermont (USA), **2009**, ranked-choice · first choices Wright 33%, Kiss 29%, Montroll 23%, Smith 15% → plurality: **Wright** · ranked-choice: **Kiss** (≈4,300 to 4,100) · one on one: **Montroll** · 2010: voters dropped ranked-choice, 52% to 48%
- Practice · trip (2 B>C>M, 2 M>C>B, 1 C>B>M) → City beats both `3–2` = Condorcet winner, but 1 first choice → **last** under plurality
- **Strategic voting**: vote for your 2nd choice to keep out the one you like least · practice: Ana's 35 + Ben's 25 → same candidate → beat Right `60–40` · two equilibria (neither gains by switching alone) · if both stick → Right wins with 40



5 Arrow's theorem

no perfect rule

- **Kenneth Arrow**, American economist (1921–2017) · book **1951** · Nobel Prize in economics **1972**
- 3+ options → no rule from rankings to a group ranking meets all four: ① everyone A > B → group A > B ② no dictator ③ works for any rankings ④ only A-vs-B rankings decide A vs B (adding C can't flip it)
- **Arrow's impossibility theorem**: "no way of turning rankings into a group ranking is free of every problem at once" ⇒ each real rule bends one condition ⇒ choosing a rule = a real choice
- Every rule has a price: plurality → split vote, a winner many rank low · runoff → can drop the Condorcet winner · one on one → sometimes none (cycle) · approval → no strength of preference
- The lesson does not say which rule is right: simplicity, a majority, or beating every rival

Watch out: "A beats B and B beats C" does **not** mean A beats C · the plurality winner can lose one on one to every rival · the median voter is the middle of the crowd, not the middle of the line · Arrow does not say every rule cycles