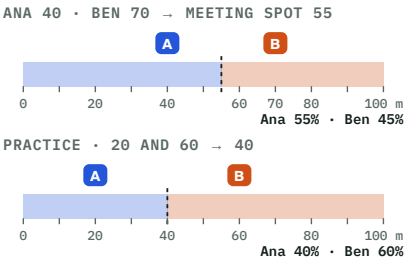
Where do two ice-cream carts stand?

Two carts on a beach, buyers walk to the nearest one. Each cart's best reply is right next to the other, on the middle side, so both end at 50: a Nash equilibrium, where nobody gains by moving alone, yet bad for the buyers.

1 The game

Hotelling's beach

- Beach **100 m** · buyers spread evenly · each walks to the **nearest cart** · carts on spots every 10 m · **your move** first (Ana's cart vs Ben at 70: never graded, never stored)
- **Meeting spot** = halfway between the carts: `(a + b) ÷ 2` · 30 and 70 → **50**
- Ana left of Ben → Ana's share = `meeting spot` · right of Ben → `100 − meeting spot` · same spot → **50 / 50** · 40 vs 70 → 55%
- **Harold Hotelling** (American economist) · "Stability in Competition", **1929**



2 Best reply

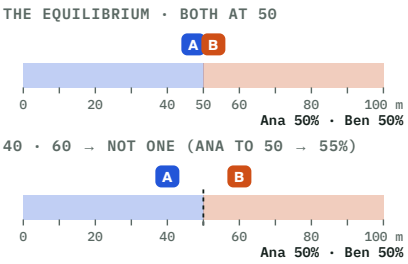
next to the rival, middle side

- **Best reply**: "the move that gives a player the best result, given what the other player does"
- Ben at 70 → Ana at **60** = **65%** (50 → 60% · on top, 70 → 50%) · then Ana at 60 → Ben hops over to **50** = **55%** (staying at 70 → 35%)
- Practice: Ben at 20 → Ana at **30** = **75%** (10 → 15% · 50 → 65%) → Ben answers at **40** = 65%
- 1,000 buyers (10 per m), $3 each · Ana 20, Ben 60 → spot 40 → **400** · Ana to 50 → spot 55 → **550** ⇒ +150 × $3 = **+$450** a day

3 Nash equilibrium

nobody moves alone

- **Nash equilibrium**: "a situation where nobody can do better by changing only their own move"
- Chase: each cart answers the other → both at **50**, 50 / 50 · Ana to 40 or 60 → only 45% ⇒ the **only** equilibrium
- Test a situation → let **one** player switch alone · one gains ⇒ **not** an equilibrium · 40 / 60 × (Ana to 50 → 55%) · 25 / 75 × (→ 62.5%) · Lesson 1 both confess ✓ · both silent × (a lone confessor goes free) · same café ✓
- **John Nash** (American mathematician) · one-page paper **1950** · Nobel Prize **1994**



4 Bad for the beach

equilibrium ≠ best

- Both at 50 → average walk **25 m** · carts at 25 and 75 → **12.5 m** (half) · same sales: 50 / 50
- Why not 25 / 75? → Ana steps to 50 → (50 + 75) ÷ 2 = **62.5%** → Ben copies → back to 50 / 50
- **No dominant strategy** (= a move best whatever the other does) here: Ana's best spot depends on Ben (60 vs 70, 50 vs 50) ⇒ equilibrium without a dominant strategy (Lesson 1 had both) · ≠ good for everyone
- No cart can fix it alone (stepping back loses buyers) → a rule from outside, e.g. spots set by the town

5 Everywhere

shops, parties, radio

- 2 coffee shops on a street → next to each other · 2 phone brands → copy each other's features
- 2 parties, voters left → right → both drift to the **median voter** ("as many voters on one side as on the other") · **Duncan Black** (Scottish economist, 1948) · **Anthony Downs** (American economist, 1957) → Lesson 10
- Practice, line 0-10: A at 2, B at 6 → spot 4 → A **40%** · A to 5 → spot 5.5 → **55%**
- Chain: nearest party → moving in wins votes → the other answers → both near the middle, alike

6 In a 2×2 table

practice: two radio stations

- 70% want pop, 30% jazz · same music → split its fans · % of listeners, **more is better**
- Pop = best reply to both (35 > 30, 70 > 15) ⇒ **dominant** for both ⇒ equilibrium **Pop / Pop, 35 / 35** · Pop / Jazz ×: Ben switches, 35 > 30 · jazz fans get no station (like the far buyers)

RADIO · % OF LISTENERS

	Ben Pop	Ben Jazz
Ana Pop	35 / 35	70 / 30
Ana Jazz	30 / 70	15 / 15

Watch out: best reply ≠ on top of the rival (→ only 50 / 50; best only when the rival is at 50) · Ana right of Ben → 100 − meeting spot · one player gaining alone is enough to break an equilibrium · equilibrium ≠ best for the buyers · an equilibrium needs no dominant strategy