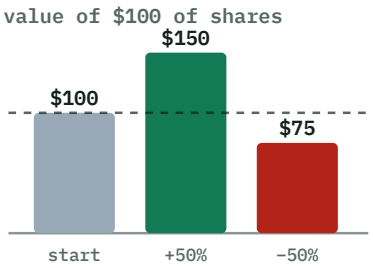
Up 50% then down 50%: back where you started?

A percent is always a percent **of** something: the value before the change. So a rise and a fall of the same percent do not cancel, two discounts do not add up, and a gap between two rates is counted in percentage points. Question 3 of the ten to ask a number.

1 Percent of what?

the value before the change

- **Percent change** = `(new − old) ÷ old × 100` → the base is always the value **before** the change
- **Coralia Gazette** (the toy island's newspaper): "stocks +50%, then −50%: back where we started"
- **Shares** (small pieces of companies): $100 → +50% of 100 → **$150** → −50% of 150 → **$75** ⇒ a quarter lost
- Second 50% is of the **bigger** number → $75 off, not $50



2 Climb back

after a fall

Fall	Left of $100	Rise needed to get back
−20%	$80	`20 ÷ 80` = **+25%**
−40%	$60	`40 ÷ 60` ≈ **+67%**
−50%	$50	`50 ÷ 50` = **+100%** (only doubling)

- **S&P 500** (index of 500 large US companies' share prices): 9 Oct 2007 **1,565** → 9 Mar 2009 **677** ≈ **−57%** → needed `1,565 ÷ 677 ≈ 2.3` ⇒ ≈ **+131%** → back above the peak 28 Mar 2013
- Deeper fall ⇒ much bigger climb back · the rise is a percent of a **smaller** number

3 Points vs percent

a gap · a share

- **Percentage point** = the gap between two percentages · **percent** = the change as a share of the old value
- **Inflation** (how fast prices rise in a year), US, BLS (US statistics office for prices and jobs): June 2022 **9.1%** → June 2023 **3.0%** ≈ 9% → 3% ⇒ **−6 points** = `6 ÷ 9` ≈ **−67%** · never "down 6%" · prices still rose, only more slowly
- Tax rate 20% → 22% ⇒ +2 points = +10% · interest 2% → 3% ⇒ +1 point = +50% · pass rate 80% → 60% ⇒ −20 points = −25% · a price ($2 → $2.50) has no points: +25%
- Both numbers true → a headline picks the one it likes → ask which

4 Changes in a row

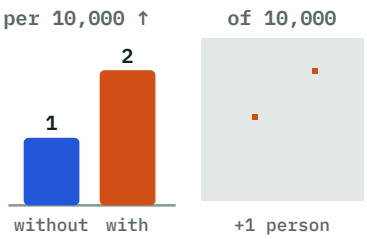
never just add them

- **Two discounts**: 20% + 20% → $100 → $80 → $64 ⇒ **36%** off · 50% + 50% → $80 → $40 → $20 ⇒ **75%**, not free · 30% + 10% → $100 → $70 → $63 ⇒ **37%**
- **Compounding** (growth on growth): 2,000 +10% → 2,200 +10% → 2,420 ⇒ **+21%**, not 20%
- **Rule of 70** (Macro Lesson 10): years to double ≈ `70 ÷ growth rate` · doubled in 10 years ⇒ ≈ 7% a year · 1.07 ten times ≈ 1.97, not 10 × 7% = 70%
- Chain: each % → of the number left by the one before → rises add up to **more**, cuts to **less**

5 Relative vs absolute

"risk doubles!"

- **Relative change** = change as % of the old value · **absolute change** = the change itself
- Made-up study of a sweetener: rash 1 in 10,000 without → 2 in 10,000 with ⇒ relative **+100%** ("doubles") · absolute **+1 person** in 10,000
- Small base ⇒ huge % · honest → give both numbers



6 So what?

question 3 of 10

- **Ten questions to ask a number** · 1 · Where does the axis start, and what is its scale? · 2 · Average of what: the mean or the median? · **3 · Percent of what? Percent or percentage points?**
- Gazette rewritten → "Shares rose 50%, then fell 50%: investors lost a quarter. They need +100% to get back." · not lying on purpose: adding the two changes sounds fair → ask: of what?

Watch out: +x% then −x% always ends **below** the start · % changes in a row never add · a fall of 6 points from 9% is not "−6%" · say the starting value and the unit: "fell 6 points, from 9% to 3%"