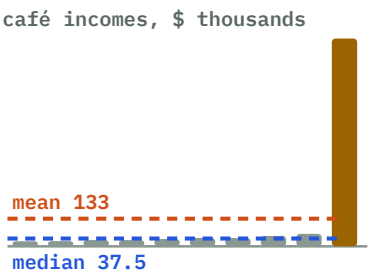
Average of what?

"Average" can mean three things: the mean, the median, the mode. One huge value pulls the mean and leaves the median alone; each one answers a different question. Question 2 of the ten to ask a number.

1 Mean

the total shared equally

- **Mean** = `total ÷ number of values` → what each gets if everything were shared equally
- **Coralia Gazette** (the toy island's newspaper): "The average Coralian earns **$133,000**"
- Café, 10 guests ($ thousands): 20 · 25 · 30 · 30 · 35 · 40 · 40 · 50 · 60 · **1,000** → total `1,330` → `1,330 ÷ 10 = 133` → **$133,000**
- Reverse → `total = mean × number` · lunch $4 × 30 students = **$120**
- Practice: villa street → `4,000 ÷ 10 = 400` · boats `49 ÷ 7 = 7` tons



2 Median

the middle of the line-up

- **Median** = middle value, all values lined up smallest → largest · half below, half above
- **Even** count → halfway between the 2 middle ones · café: 5th and 6th = 35, 40 → `(35 + 40) ÷ 2 = 37.5` → **$37,500** · villa street `(180 + 200) ÷ 2 = 190`
- **Odd** count → the middle one · boats `2 3 3 4 5 6 26` → **4** (the 4th of 7)

3 Outlier

pulls only the mean

- **Outlier** = value far from all the others (the café's $1,000,000, the $2,350,000 villa)
- Tenth guest $1,000,000 → $60,000 ⇒ mean **$133,000 → $39,000** · median **$37,500 → $37,500**
- Villa 2,350 → 300 ⇒ mean 400 → 195 · median 190 → 190 ($ thousands)
- Town of 1,000, mean $40,000 + one person at $20M ⇒ total $40M → $60M ⇒ mean ≈ **$60,000 (+50%)** · median ≈ same · typical family: same
- Chain: outlier in → total ↑ → mean ↑ → median ≈ same

4 Which one?

mean · median · mode

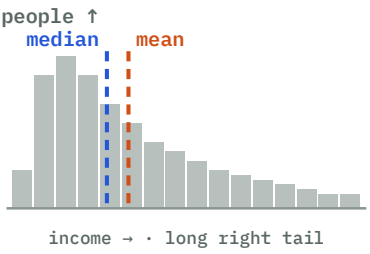
MEAN → totals	MEDIAN → the typical case	MODE → the most common
a 1% tax on all incomes · a class's test score vs another class · loaves per baker → bakery total · lunch bill · bus fuel	typical family income · house price a typical buyer pays · wage half earn less than · typical rent · typical pay at a firm with a $5M boss	shoe size to stock · T-shirt size to order · café: 30 and 40 tie (twice each) · boats: 3

- **Mode** = the value that appears most often · rarely useful for incomes
- **Median's blind spot**: 5 workers 20 · 30 · 40 · 50 · 60 → 20 · 30 · 40 · **70** · **90** ⇒ median 40 → 40 · mean `200 ÷ 5 = 40` → `250 ÷ 5 = 50` ⇒ "nobody better off" is false → report both

5 Real world

long right tail

- **US Census Bureau** (the US statistics office for people and households), 2024 household income: median **$83,730** · mean **$121,000** → `121,000 ÷ 83,730 ≈ 1.45` → mean ≈ 45% higher
- **Federal Reserve** (US central bank), net worth = owned − owed, late 2025: richest 1% ≈ **32%** · poorer half ≈ **2.5%**
- **Histogram** = how many people at each income → most on the left, a few very high incomes → **long right tail** ⇒ mean pulled toward the tail, median where half are on each side
- **GDP per person** (Macro Lesson 1) = GDP ÷ population = a **mean** → hides how income is shared



6 So what?

question 2 of 10

- **Ten questions to ask a number** · 1 · Where does the axis start, and what is its scale? · **2 · Average of what: the mean or the median?**
- Gazette rewritten → "The **mean** Coralian earns $133,000; the **median** Coralian earns $37,500." · not wicked: "average" has three meanings → ask which

Watch out: "average" in the news is usually the **mean** · mean far above median ⇒ a few very high values · median of an even count = halfway between the 2 middle ones, not the bigger one · the median cannot see changes away from the middle